Eur. Phys. J. A **12**, 361–367 (2001)

THE EUROPEAN PHYSICAL JOURNAL A

Resonance-reggeon and parton-hadron duality in strong interactions

L. Jenkovszky[1,3], V.K. Magas[1,2,a], and E. Predazzi[3]

[1] Bogolyubov Institute for Theoretical Physics, Academy of Sciences of Ukraine, Metrologicheskaya str. 14b, 01143 Kiev, Ukraine
[2] Center for Physics of Fundamental Interactions, Instituto Superior Tecnico, Av. Rovisco Pais, 1049-001 Lisbon, Portugal
[3] Dipartimento di Fisica Teorica, Università di Torino, sezione INFN di Torino, via P. Giuria, 1, Torino, 10125 Italy

Received: 30 October 2001
Communicated by V.V. Anisovich

Abstract. By using the concept of duality between direct channel resonances and Regge exchanges we relate the small- and large-x behavior of the structure functions. We show that even a single resonance exhibits Bjorken scaling at large Q^2.

PACS. 12.40.Nn Regge theory, duality, absorptive/optical models – 13.60.Hb Total and inclusive cross sections (including deep-inelastic processes) – 11.55.Bq Analytic properties of S matrix – 11.55.Hx Sum rules

1 Introduction

Inspired by recent experimental measurements of the nucleon structure functions at the JLab (CEBAF) [1], we suggest a unified "two-dimensionally dual" picture of the strong interaction [2–4] connecting low and high energies (Veneziano, or resonance-reggeon duality [5]) with low and high virtualities (Q^2) (Bloom-Gilman, or hadron-parton duality [6]). The basic idea of the unification is the use of a Q^2-dependent dual amplitudes, employing nonlinear complex Regge trajectories providing an imaginary part of the scattering amplitude, related to the total cross-section and structure functions and thus saturating duality by a finite number of resonances lying on the (limited) real part of the Regge trajectories.

The resulting object, a deeply virtual scattering amplitude, $A(s,t,Q^2)$, is a function of three variables, reducing to a nuclear-structure function (SF) when $t=0$ and to an on-shell hadronic scattering amplitude for $Q^2=m^2$. It closes the circle in fig. 1. We use this amplitude to describe the background as well as the resonance component [7].

The Q^2-dependence of the residuae functions here will be chosen in such a way as to provide for Bjorken scaling at small x (large s). The resulting amplitude (structure function) is applicable in the whole kinematical range, including the resonance region. We call this unification "two dimensional duality" —one in s, the other one in Q^2.

In the early days of duality, off mass continuation was attempted [8] by means of multileg (*e.g.*, 6-point) dual amplitudes with "extra" lines taken at their poles. Without going into details, here we only mention that scaling in this approach can be achieved [9] only with nonlinear trajectories, *e.g.* those with logarithmic or constant asymptotic.

[a] e-mail: vladimir@cfif.ist.utl.pt

2 Notation and conventions

We use standard notation for the cross-section and structure function (see fig. 2):

$$\sigma^{\gamma^* p} = \frac{4\pi^2\alpha(1+4m^2x^2/Q^2)}{Q^2(1-x)}\frac{F_2(x,Q^2)}{1+R(x,Q^2)}\,, \qquad (1)$$

where α is the fine structure constant, Q^2 is minus the squared four-momentum transfer or the momentum carried by the virtual photon, x is the Bjorken variable and s is the squared center-of-mass energy of the $\gamma^* p$ system, obeying the relation

$$s = Q^2(1-x)/x + m_p^2\,, \qquad (2)$$

where m_p is the proton mass and $R(x,Q^2) = \sigma_L(x,Q^2)/\sigma_T(x,Q^2)$. For the sake of simplicity we set $R=0$, which is a reasonable approximation.

We use the norm where

$$\sigma_T^{\gamma^*}(s,t,Q^2) = \mathrm{Im}\ A(s,t,Q^2)\,. \qquad (3)$$

According to the two-component duality picture [7], both the scattering amplitude A and the structure function F_2

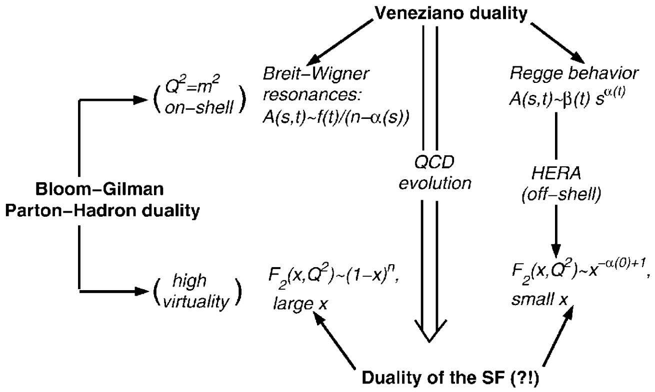

Fig. 1. Veneziano, or resonance-reggeon duality [5] and Bloom-Gilman, or hadron-parton duality [6] in strong interactions.

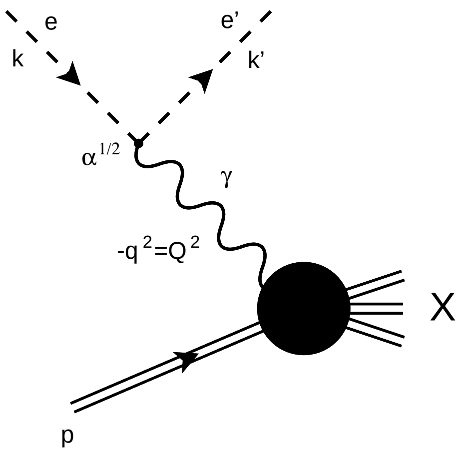

Fig. 2. Kinematics of deep inelastic scattering.

are sums of diffractive and nondiffractive terms. At high energies both terms are Regge behaved. In $\gamma^* p$ scattering only positive-signature exchanges are allowed. The dominant ones are the pomeron and the f reggeon, respectively. The relevant scattering amplitude is (remember that here $t = 0$)

$$A_i(s, Q^2) = i\beta_k(Q^2)\Big(-i\frac{s}{s_i}\Big)^{\alpha_k(0)-1}, \tag{4}$$

where α and β are the Regge trajectory and residue and k stands either for the pomeron or the reggeon. As usual, the residue will be chosen such as to satisfy approximate Bjorken scaling for the structure function [10,11]. It should be remembered that by factorization, assuming that the reggeon (or pomeron) exchange is a simple pole, the residue function is a product of two vertices —the $\gamma\gamma R(P)$ and $NNR(P)$, where N stands for the nucleon (see fig. 3).

At low energies the scattering amplitude is dominated by the contribution of the near-by resonances. In the vicinity of a resonance Res, the amplitude can be also written in a factorized form — as aproduct of the probabilities that two particles, γ and p, form a resonance with the squared mass s_{Res} and total width Γ:

$$A(s, Q^2) = \sum_{\text{spin}} \frac{A_{fi}(Q^2)A^*_{if}(Q^2)}{s_{Res} - s - i\Gamma}\,, \tag{5}$$

where A_{fi} are the inelastic form factors.

3 Nucleon resonances in inelastic electron-nucleon scattering

Some thirty years ago Bloom and Gilman [6] observed that the prominent resonances in inelastic electron-proton scattering do not disappear with increasing Q^2 relatively to the "background" but instead fall at roughly the same rate as any background. Furthermore, the smooth scaling limit proved to be an accurate average over resonance bumps seen at lower Q^2 and s.

Since then, the phenomenon was studied in a number of papers [12,13] and recently has been confirmed experimentally [1]. These studies were aimed mainly to answer the question: in which way a limited number of resonances can reproduce the smooth scaling behavior? The main theoretical tools in these studies were the finite-energy sum rules and perturbative QCD calculations, whenever applicable. Our aim instead is the construction of an explicit dual model combining direct channel resonances, Regge behavior, typical of hadrons and scaling behavior, typical for the partonic picture.

The existence of resonances in the structure function at large x, close to $x = 1$ by itself is not surprising: by the relations (1) and (2) they are the same as in $\gamma^* p$ total cross-section, but in a different coordinate system. The important question is whether and, if so, how a small number of resonances (or even a single one) can reproduce the smooth Bjorken scaling behavior, known to be an asymptotic property, typical of multiparticle processes.

The possibility that a limited (small) number of resonances can build up smooth Regge behavior was demonstrated by means of finite-energy sum rules [14]. Later it was confused by the presence of an infinite number of narrow resonances in the Veneziano model [5], which made its phenomenological application difficult, if not impossible. Similar to the case of the resonance-reggeon duality [14], hadron-parton duality was established [6] by means of the finite-energy sum rules, but it was not realized explicitly like the Veneziano model (or its further modifications).

Actually, the early onset of Bjorken scaling, called "early, or precaution scaling" was observed with the first measurements of deep inelastic scattering at SLAC, where it was noticed that a more rapid approach to scaling can be achieved with the Bloom-Gilman (BG) variable [6] $x' = x/(1 + m^2x^2/Q^2)$ instead of x (or $\omega = 1/x$). More recently the following generalization of the BG variable:

$$\xi = \frac{2x}{1 + \sqrt{1 + \frac{4m^2x^2}{Q^2}}}\,, \tag{6}$$

was suggested by O. Nachtmann [15]. We use the standard Bjorken variable x, however our results can be easily

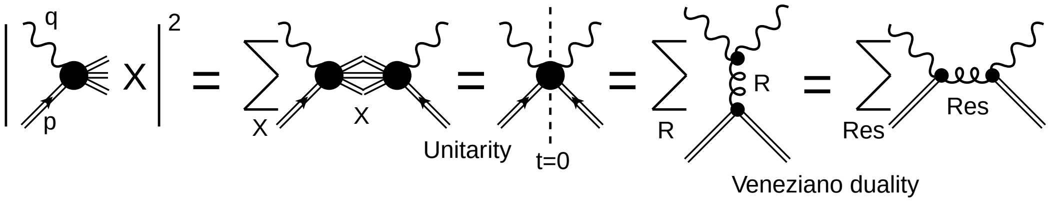

Fig. 3. According to the Veneziano (or resonance-reggeon) duality a proper sum of either t channel or s channel resonance exchanges accounts for the whole amplitude.

rewritten in terms of the above-mentioned modified variables.

First attempts to combine resonance (Regge) behavior with Bjorken scaling were made [16–18] at low energies (large x), with the emphasis on the right choice of the Q^2-dependence, such as to satisfy the required behavior of the form factors, vector meson dominance (VMD) with the requirement of the Bjorken scaling. (N.B.: the validity (or failure) of the (generalized) VMD is still disputed.) Similar attempts in the high-energy (low x) region became popular recently, with the advent of the HERA data. They will be presented in sect. 5.

A consistent treatment of the problem requires the account for the spin dependence. For simplicity, we ignore it in this paper (see, *e.g.* [13]).

4 Factorization and dual properties (bootstrap) of the vertices

Since the purpose of the present paper is the construction of a unified model realizing duality both in the s and t channels, we first attempt to identify its fragments, namely the vertices (to be interpreted later on as Q^2-dependent form factors).

Let us remind that the residue functions are completely arbitrary in the Regge pole model, but they are constrained in the dual model. We show this by using the low-energy (resonances) and high-energy (Regge) decomposition on the simple Veneziano model [5]:

$$V(s,t)=\int_0^1 \mathrm{d}z z^{-\alpha(s)}(1-z)^{-\alpha(t)}=$$
$$B(1-\alpha(s),1-\alpha(t))=\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}\,. \quad (7)$$

Furthermore,

$$V(s,t)=\sum_{n=1}^{\infty}\frac{1}{n-\alpha(s)}\frac{\Gamma(n+\alpha(t)+1)}{n!\;\Gamma(\alpha(t)+1)}\,. \quad (8)$$

By the Stirling formula

$$V(s,t)\Bigg|_{|\alpha(s)|\to\infty}\to[-\alpha(s)]^{\alpha(t)-1}\Gamma\big(1-\alpha(t)\big)$$
$$\times\left[\sum_{n=0}^{N}\frac{a_n(0)}{[\alpha(s)]^n}+0\left(\frac{1}{[\alpha(s)]^{N+1}}\right)\right] \quad (9)$$

and since for small $|t|$ the Γ function varies slowly compared with the exponential, the Regge asymptotic behavior is

$$V(s,t)\sim(\alpha' s)^{\alpha(t)}\,, \quad (10)$$

where $\beta(t)=(\alpha')^{\alpha(t)}$ is the Regge residue.

Actually, one has to identify a single (and hence economic!) Regge exchange amplitude with a sum of direct channel poles. Such an identification is not practical for an infinite number of poles (*e.g.*, the Veneziano amplitude) but, as we show below is feasible if their number is finite (small). To anticipate the forthcoming discussion, we shall feed the Q^2-dependence in the Regge residue at high energies (small x and use the dual amplitude with a finite number of resonances!) to the whole kinematical region, including that of resonances. Relating the amplitude to the SF, we set $t=0$.

To remedy the problems of the infinite number of narrow resonance, nonunitarity and lack of an imaginary part, we use a generalization of the Veneziano model free from the above-mentioned difficulties.

5 Dual amplitude with Mandelstam analyticity

The invariant dual on-shell scattering amplitude with Mandelstam analyticity (DAMA) applicable both to the diffractive and nondiffractive components reads [19]

$$D(s,t)=\int_0^1 \mathrm{d}z\left(\frac{z}{g}\right)^{-\alpha(s')-1}\left(\frac{1-z}{g}\right)^{-\alpha(t')-1}, \quad (11)$$

where $s'=s(1-z)$, $t'=tz$, g is a parameter, $g>1$, and s, t are the Mandelstam variables.

For $s\to\infty$ and fixed t it has the following Regge asymptotic behavior

$$D(s,t)\approx\sqrt{\frac{2\pi}{\alpha_t(0)}}g^{1+a+ib}\left(\frac{s\alpha'(0)g\ln g}{\alpha_t(0)}\right)^{\alpha_t(0)-1}, \quad (12)$$

where $a=\mathrm{Re}\;\alpha\Big(\frac{\alpha_t(0)}{\alpha'(0)\ln g}\Big)$ and $b=\mathrm{Im}\;\alpha\Big(\frac{\alpha_t(0)}{\alpha'(0)\ln g}\Big)$.

The pole structure of DAMA is similar to that of the Veneziano model except that multiple poles may appear at daughter levels. The presence of these multipoles does not contradict the theoretical postulates. On the other hand, they can be removed without any harm to the dual model by means of the so-called Van der Corput neutralizer. The procedure [19] is to multiply the integrand of (11) by a function $\phi(x)$ with the properties:

$$\phi(0)=0, \quad \phi(1)=1, \quad \phi^n(1)=0, \quad n=1,2,3,...$$

The function $\phi(x)=1-\exp\left(-\frac{x}{1-x}\right)$, for example, satisfies the above conditions and results [19] in a standard, "Veneziano-like" pole structure:

$$D(s,t)=\sum_n g^{n+\alpha_t(0)}\frac{C_n}{n-\alpha(s)}\,, \tag{13}$$

where

$$C_n=\frac{\alpha_t(0)(\alpha_t(0)+1)...(\alpha_t(0)+n+1)}{n!}\,. \tag{14}$$

The pole term (13) is a generalization of the Breit-Wigner formula (5), comprising a whole sequence of resonances lying on a complex trajectory $\alpha(s)$. Such a "reggeized" Breit-Wigner formula has little practical use in the case of linear trajectories, resulting in an infinite sequence of poles, but it becomes a powerful tool if complex trajectories with a limited real part and hence a restricted number of resonances are used. Moreover, it appears that a small number of resonances is sufficient to saturate the direct channel.

Contrary to the Veneziano model, DAMA (11) not only allows but rather requires the use of nonlinear complex trajectories providing, in particular, for the imaginary part of the amplitude, resonance widths and resulting in a finite number of those. More specifically, the asymptotic rise of the trajectories in DAMA is limited by the condition (in accordance with an important upper bound derived earlier [20])

$$\left|\frac{\alpha(s)}{\sqrt{s}\ln s}\right|\le \text{const}\,, \quad s\to\infty\,. \tag{15}$$

Actually, this upper bound can be even lowered up to a logarithm by requiring wide-angle scaling behaviour for the amplitude.

Models of Regge trajectories combining the correct threshold and asymptotic behaviors have been widely discussed in the literature (see, *e.g.* [21] for a recent treatment of this problem). A particularly simple model is based on a sum of square roots

$$\alpha(s)=\alpha_0+\sum_i\gamma_i\sqrt{s_i-s}\,,$$

where the lightest threshold (made of two pions or a pion and a nucleon) is important for the imaginary part, while the heaviest threshold limits the rise of the real part, where resonances terminate.

Dual amplitude with Mandelstam analyticity with the trajectories specified above is equally applicable to both: the diffractive and nondiffractive components of the amplitude, the difference being qualitative rather than quantitative. The utilization of a trajectory with a single threshold,

$$\alpha_E(s)=\alpha_E(0)+\alpha_{1E}(\sqrt{s_E}-\sqrt{s_E-s})\,, \tag{16}$$

prevents the production of resonances on the the physical sheet, although they are present on the nonphysical sheet, sustaining duality (*i.e.*, their sum produces Regge asymptotic behavior). This nontrivial property of DAMA makes it particularly attractive in describing the smooth background (dual to the pomeron exchange) (see [19]). The threshold value, slope and the intercept of this exotic trajectory are free parameters.

For the resonance component a finite sum in (13) is adequate, but we shall use a simple model with lowest threshold included explicitly and the higher ones approximated by a linear term

$$\alpha_R(s)=\alpha_R(0)+\alpha' s+\alpha_{1R}(\sqrt{s_0}-\sqrt{s_0-s}), \tag{17}$$

where s_0 is the lowest threshold —$s_0=(m_\pi+m_p)^2$ in our case— while the remaining 3 parameters will be adjusted to the known properties of the relevant trajectories (N^* and Δ isobar in our case). The termination of resonances, provided in DAMA by the limited real part, here will be effectively taken into account by a cutoff in the summation of (13).

Finally, we note that a minimal model for the scattering amplitude is a sum

$$A(s,t,u)=c(D(s,t)+D(u,t)), \tag{18}$$

providing the correct signature at high-energy limit, c is a normalization factor. We disregard the symmetry (spin and isospin) properties of the problem, concentrating on its dynamics. In the limit $s\to\infty,\quad t=0$ we have $u=-s$ and therefore

$$A(s,0,-s)|_{s\to\infty}=c\;D(s,0)(1+(-1)^{\alpha_t(0)-1}), \tag{19}$$

where $D(s,t)$ is given by eq. (12). For the total cross-section in this limit we obtain

$$\begin{aligned}\sigma_T^{\gamma^*}&=\text{Im}\;A=Cg^{\alpha_t(0)+a}(s\alpha'(0)\ln g)^{\alpha_t(0)-1}\\&\cdot(\sin(\alpha_t(0)-1)\pi\cos(b\ln g)\\&+(1+\cos(\alpha_t(0)-1)\pi)\sin(b\ln g))\,,\end{aligned} \tag{20}$$

where C is a constant independent of s, g and $\alpha'(0)$.

6 Q^2-dependence

Our main idea is to introduce the Q^2-dependence in the dual model by matching its Regge asymptotic behavior and pole structure to standard forms, known from the literature. The point is that the correct identification of this Q^2-dependence in a single asymptotic limit of the dual amplitude will extend it to the rest of the kinematical regions. We have two ways to do so:

a) combine Regge behavior and Bjorken scaling limits of the structure functions (or Q^2-dependent γ^*p cross-sections);
b) introduce properly the Q^2-dependence in the resonance region.

They should match if the procedure is correct and the dual amplitude should take care of any further inter or extrapolation.

It is obvious from eq. (4) that asymptotic Regge and scaling behavior require the residue to fall like $\sim (Q^2)^{-\alpha_i(0)+1}$. Actually, it could be more involved if we require the correct $Q^2 \to 0$ limit to be respected and the observed scaling violation (the "HERA effect") to be included. Various models to cope with the above requirements have been suggested [10,11,22]. At HERA, especially at large Q^2, scaling is so badly violated that it may not be explicit anymore.

In combining Regge asymptotic behavior with (approximate) Bjorken scaling, one can proceed basically in the following way: keep explicitly a scaling factor x^{Δ} (to be broken by some Q^2-dependence "properly" taken into account) [11]

$$F_2(x,Q^2) \sim x^{-\Delta(Q^2)}\Big(\frac{Q^2}{Q^2+Q_0^2}\Big)^{1+\Delta(Q^2)}, \qquad (21)$$

where $\Delta(Q^2) = \alpha_t(0) - 1$ may be a constant, in particular.

Note that since the Regge asymptotic of the Veneziano model is $\sim (-\alpha' s)^{\alpha(t)-1}$, the only way to incorporate there Q^2-dependence is through the slope α' [2,3], *i.e.* by making the trajectories Q^2-dependent, thus violating Regge factorization. Q^2-dependent intercepts were used earlier [10,11] in a different context, namely to cope with the observed "hardening" of small-x physics with increasing Q^2 (Bjorken scaling violation). Although we do not exclude this possibility (treating it as "effective" Regge pole, we study here the different option of introducing scaling violation in the constant g appearing, besides α', in the residue of DAMA, eq (11).

From the explicit Regge asymptotic form of DAMA, (20), and neglecting the logarithmic dependence of g we make the following identification:

$$g(Q^2)^{\alpha_t(0)+a} = \left(\frac{Q^2_{\rm lim}}{Q^2+Q_0^2}\right)^{\alpha_t(0)}. \qquad (22)$$

Note that eq. (22) is transcendent with respect to g, since $a = a(g) = \text{Re } \alpha\Big(\frac{\alpha_t(0)}{\alpha'(0)\ln g}\Big)$. Another point to mention is that this equation is not valid in the whole range of Q^2, since for Q^2 close to $Q^2_{\rm lim}$, g may get smaller than 1, which is unacceptable in DAMA. For large Q^2 the Q^2-dependence of the $\log g$ and $b = b(Q^2) = \text{Im } \alpha\Big(\frac{\alpha_t(0)}{\alpha'(0)\ln g}\Big)$ in eq. (20) cannot be neglected, it might contribute to scaling violation.

7 Scaling at large x

Let us now consider the extreme case of a single-resonance contribution.

A resonance pole in DAMA contributes with

$$A(s,t) = g^{n+\alpha_t(0)}\frac{C_n}{n-\alpha(s)}.$$

At the resonance $s = s_{Res}$ one has Re $\alpha(s_{Res}) = n$ and $\frac{Q^2(1-x)}{x} = s_{Res} - m^2$, hence

$$F_2(x,Q^2) = \frac{Q^2(1-x)}{4\pi^2\alpha\Big(1+\frac{4m^2x^2}{Q^2}\Big)}\frac{C_n}{\text{Im }\alpha(s_{Res})}g(Q^2)^{n+\alpha_t(0)}.$$

As $x \to 1$ $Q^2 \approx \frac{s_{Res}-m^2}{1-x} \to \infty$ and

$$F_2(x,Q^2) \sim g\Big(\frac{s_{Res}-m^2}{1-x}\Big)^{n+\alpha_t(0)}.$$

By using the approximate solution $g(Q^2) \approx \big(Q^2_{\rm lim}/Q^2\big)^{\frac{\alpha_t(0)}{\alpha_t(0)+a}}$, where a is a slowly varying function of g, we get for x near 1

$$F_2(x,Q^2) \sim (1-x)^{\frac{\alpha_t(0)(n+\alpha_t(0))}{\alpha_t(0)+a}},$$

where the limits for x are defined by $Q_0^2 \ll \frac{s_{Res}-m^2}{1-x} \le Q^2_{\rm lim}$.

We recognize a typical large-x scaling behavior $(1-x)^N$ with the power N (counting the quarks in the reaction) depending basically on the intercept of the t channel trajectories.

8 Numerical estimates

Having fixed the Q^2-dependence of the dual model by matching its Regge asymptotic behavior with that of the structure functions, we now use this dual model to extrapolate down to the resonance region, where its pole expansion (13) is appropriate —now complemented with a Q^2-dependence through $g(Q^2)$, fixed by eq. (22).

As already said, we write the imaginary part of the scattering amplitude as the sum of two terms —a diffractive (background) and nondiffractive (resonance) one. Note that $g(Q^2)$ has the same functional form (22) in both cases, only the values of the parameters differ (they are fixed from the small-x fits [22] of the SF).

At low, resonance, energies $\gamma^* p$ scattering exhibits a rich resonance structure, intensively studied in a number of papers. About 20 resonances overlap, their relative importance varying with Q^2, but only a few can be identified more or less unambiguously. These are: $\Delta^+(1236)$ with $J^P = \frac{3}{2}^+$, $N^{*+}(1520)$, $J^P = \frac{3}{2}^-$, $N^{*+}(1688)$, $J^P = \frac{5}{2}^+$ and $N^{*+}(1920)$ with $J^P = \frac{7}{2}^+$. They lie on the Δ and the exchange-degenerate N trajectories. In this work we are mainly interested in introducing Q^2-dependence into the scattering amplitude, therefore we will concentrate on a single resonance ($\Delta^+(1236)$) at different values of Q^2. We use trajectories (17) in which the lowest pion-nucleon threshold is included explicitly, while higher thresholds are approximated by a linear term

$$\alpha_\Delta(s) = 0.1 + 0.84s + 0.1331(\sqrt{s_0} - \sqrt{s_0 - s}),$$

where $s_0 = (m_\pi^2 + m_N^2)^1$. The above values of the parameters are chosen so as to fit the known mass and width of

[1] Actually, trajectories without any linear term (see, *e.g.* [21]) could be more appropriate (and will be studied in the future).

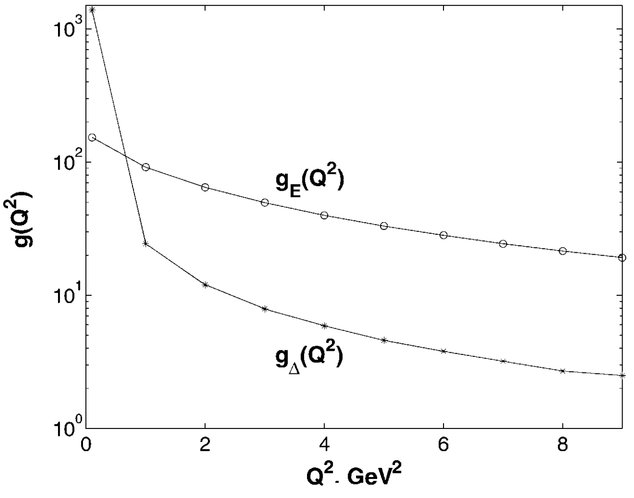

Fig. 4. $g(Q^2)$ —the solution of the transcendent equation (22)— for the Δ and the exotic trajectories.

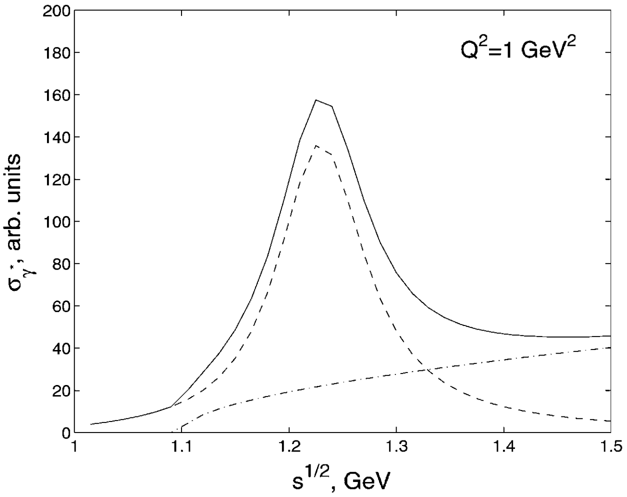

Fig. 5. $\gamma^* p$ total cross-section as a function of $\sqrt{s}$. The dashed line shows the contribution from the Δ resonance, the dot-dashed line corresponds to the background, *i.e.* the contribution from the exotic trajectory. Here $Q^2 = 1\,\mathrm{GeV}^2$.

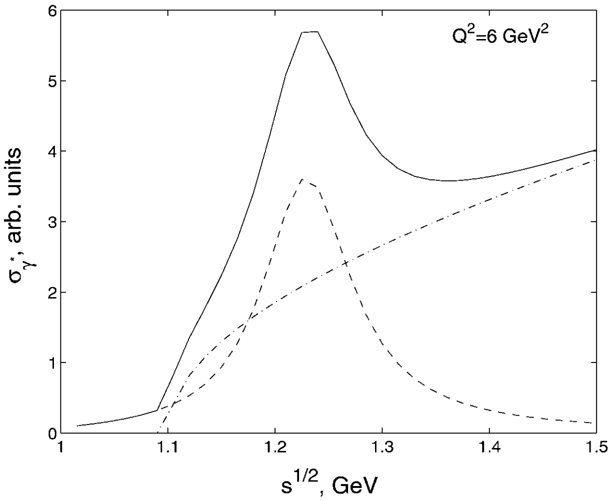

Fig. 6. $\gamma^* p$ total cross-section as a function of $\sqrt{s}$. The dashed line shows the contribution from the Δ resonance, the dot-dashed line corresponds to the background, *i.e.* the contribution from the exotic trajectory. Here $Q^2 = 6\,\mathrm{GeV}^2$.

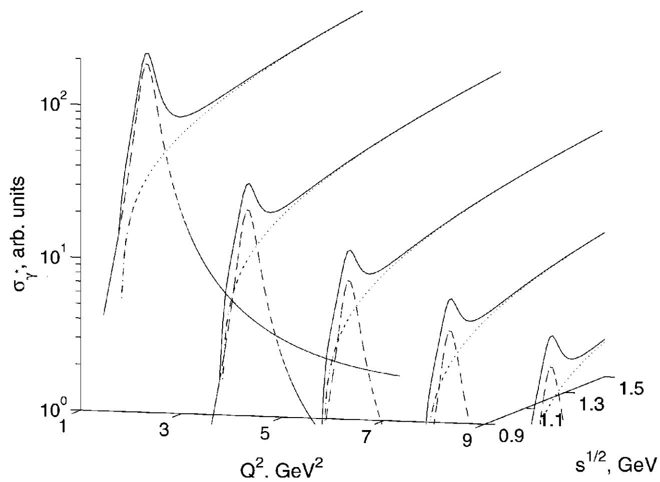

Fig. 7. $\gamma^* p$ total cross-section as a function of $\sqrt{s}$ and Q^2. For different values of Q^2 we show the contributions from the Δ resonance (dashed line), the background, *i.e.* the contribution from the exotic trajectory (dot-dashed line) and their sum (full line).

the Δ resonance in a way consistent with the known linear parameterizations.

In the interval of interest $\sqrt{s} = 1.1$–$1.5\,\mathrm{GeV}$, $t = 0$, we have $u = m_N^2 - s < 0$, so, it is far from resonance region, therefore we neglect the contribution from $D(u,t)$ for both resonance and background terms.

The smooth background is also modeled by a single term and exotic trajectory (16). As already explained, the direct channel Regge pole does not produce here physical resonances. The parameters of the exotic trajectory are

$$\alpha_E(s) = -0.25 + 0.25(\sqrt{1.21} - \sqrt{1.21 - s}), \tag{23}$$

where $s_{\mathrm{E}} = 1.1^2\,\mathrm{GeV}^2$ is an effective exotic threshold. Obviously "pole" does not mean a resonance in this case.

Figure 4 shows g as a function of Q^2 for Δ and exotic trajectories. The resulting cross-sections (imaginary part of the amplitude) in the resonance region is shown in figs. 5 and 6 for two values of $Q^2 = 1$ and $6\,\mathrm{GeV}^2$. It is in qualitative agreement with the experimental data [4]. Figure 7 shows the dual properties of the cross-section in 2 dimensions —one is the squared energy s and the other one is virtuality Q^2. Table 1 shows the values of the parameters used in our calculations.

The main conclusions from our analysis are that:

a) the Q^2-dependence at low and high x (or high and low s) are interrelated and have the same origin;
b) even a single (low energy) resonance can produce a smooth scaling-like curve in the structure function (parton-hadron duality).

To summarize, we have suggested an explicit dual model in which the Q^2-dependence introduced in the low-x domain

Table 1. Parameters used in the calculations shown in figs. 4, 5 and 6. (Normalization coefficient $c = 0.03$.)

	Δ Resonance	Background
$Q^2_{\rm lim}$ (GeV^2)	62	120
Q^2_0 (GeV^2)	0.01	2.5
Dual trajectory	$\alpha_f(t)$ is dual to $\alpha_\Delta(s)$	$\alpha_P(t)$ is dual to $\alpha_E(s)$
	$\alpha_f(0) = 0.9$	$\alpha_P(0) = 1 + 0.077 \cdot \left(1 + \frac{2Q^2}{Q^2+1.117}\right)$ [11]

is extended to the whole kinematic region, in particular to the region of resonances. The resulting predictions for the first resonance in the $\gamma^* p$ system shown in figs. 5, 6 are in quantitative agreement with data.

L.J. thanks INFN and the Torino University, where this work was completed, for their hospitality and support. L.J. and V.M. acknowledge the support by INTAS, Grant 00-00366. The work of L.J. was supported also by the US Civilian Research and Development Foundation (CRDF), Grant UP1-2119.

References

1. I. Niculescu et al., Phys. Rev. Lett. **85**, 1182; 1186 (2000).
2. L.L. Jenkovszky, V.K. Magas, F. Paccanoni, *Proceedings of the New Trend in High-Energy Physics, Crimea, Ukraine, May 27-June 4, 2000*, edited by P.N. Bogolyubov, L.L. Jenkovsky (Bogolyubov Institute for Theoretical Physics, Kiev, 2000) p. 121.
3. R. Fiore, L. Jenkovszky, V. Magas, *Proceedings of the Diffractin-2000, Cetraro, Italy, 2-8 September 2000*, Nucl. Phys. B Proc. Suppl. **99**, 131 (2001).
4. L. Jenkovszky, V. Magas, to be published in *Proceedings of the 9th Blois Workshop On Elastic and Diffractive Scattering, June 9-15, 2001, Prague, Czech Republic*; to be published in *Proceedings of Spin-2001, 9th International Workshop On High-Energy Spin Physics, 2-7 Aug 2001, Dubna, Russia*; to be published in *Proceedings of the ISMD 2001, XXXI International Symposium on Multiparticle Dynamics, September 1-7, 2001, Datong, China*, hep-ph/0111398; L. Jenkovszky, T. Korzhinskaya, V. Kuvshinov, V. Magas, to be published in *Proceedings of the New Trend in High-Energy Physics, Yalta, Crimea, Ukraine, September 22-29, 2001*; L. Jenkovszky, V.K. Magas, E. Predazzi, to be published in *Proceedings of the 6th International Summer School-Seminar On Actual Problems Of High Energy Physics, Gomel, Belarus, August 7-16, 2001*, nucl-th/0110085.
5. G. Veneziano, Nuovo Cimento A **57**, 190 (1968).
6. E.D. Bloom, E.J. Gilman, Phys. Rev. Lett. **25**, 1149 (1970); Phys. Rev. D **4**, 2901 (1971).
7. P. Freund, Phys. Rev. Lett. **20**, 235 (1968); H. Harari, Phys. Rev. Lett. **20**, 1395 (1968).
8. V. Rittenberg, H.R. Rubinstein, Nucl. Phys. B **28**, 184 (1971);
9. G. Schierholz, M.G. Schmidt, Phys. Rev. D **10**, 175 (1974).
10. M. Bertini, M. Giffon, E. Predazzi, Phys. Lett. B **349**, 561 (1995).
11. A. Capella, A. Kaidalov, C. Merino, J. Tran Thanh Van, Phys. Lett. B **337**, 358 (1994); L.P.A. Haakman, A. Kaidalov, J.H. Koch, Phys. Lett. **365**, 411 (1996).
12. A. De Rujula, H. Georgi, H.D. Politzer, Ann. Phys. (N.Y.) **103**, 315 (1977); P. Stoler, Phys. Rev. Lett. **66**, 1003 (1991); P. Stoler, Phys. Rev. D **44**, 73 (1991); I. Afanasiev, C.E. Carlson, Ch. Wahlqvist, Phys. Rev. D **62**, 074011 (2000); F.E. Close, N. Isgur, Phys. Lett. B **509**, 81 (2001); N. Isgur, S. Jeschonnek, W. Melnitchouk, J.W. Van Orden, Phys. Rev. D **64**, 054004 (2001).
13. C.E. Carlson, N. Mukhopadhyay, Phys. Rev. D **41**, 2343 (1990).
14. A.A. Logunov, L.D. Soloviov, A.N. Tavkhelidze, Phys. Lett. B **24**, 181 (1967); R. Dolen, D. Horn, C. Schmid Phys. Rev. **166**, 1768 (1968).
15. O. Nachtmann, Nucl. Phys. B **63**, 237 (1973); **78**, 455 (1974).
16. Marc Damashek, Frederick J. Gilman, Phys. Rev. D **1**, 1319 (1970).
17. A. Bramon, E. Etim, M. Greco, Phys. Lett. B **41**, 609 (1972).
18. E. Etim, A. Malecki, Nuovo Cimento A **104**, 531 (1991).
19. A. Bugrij et al., Fortschr. Phys. **21**, 427 (1973).
20. A. Degasperis, E. Predazzi, Nuovo Cimento A **65**, 764 (1970).
21. R. Fiore, L. Jenkovszky, V. Magas, F. Paccanoni, A. Papa, Eur. Phys. J. A **10**, 217 (2001), hep-ph/0011035.
22. L. Jenkovszky, E. Martynov, F. Paccanoni, Padova preprint PFDPD 95/TH/21; P. Desgrolard, A.I. Lengyel, E.S. Martynov, Eur. Phys. J. C **7**, 655 (1999).